10. cvičení - výsledky

Příklad 1.

(a) $\frac{1}{2}\log|\cos x-1|-\frac{1}{2}\log|\cos x+1|$ pro $x\in(k\pi,(k+1)\pi), k\in\mathbb{Z}$.

(b) $\frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\tan x+1\right)+\frac{1}{\sqrt{2}}\arctan\left(\sqrt{2}\tan x-1\right)$.

(c) $\frac{1}{2}\sin^2 x+2\sin x+3\log(2-\sin x)$.

(d) $\frac{2}{\sqrt{3}}\arctan(\sqrt{3}\tan\frac{x}{2})$.

(e) $\frac{1}{\sqrt{2}}\arctan(\sqrt{2}\tan x)$.

(f) $\log|\tan x|-\frac{1}{2\sin^2 x}$ pro $x\in(k\frac{\pi}{2},(k+1)\frac{\pi}{2}), k\in\mathbb{Z}$.

(g) $x+\frac{2}{1+\tan\frac{x}{2}}$ pro $x\in(-\frac{\pi}{2}+k2\pi,\frac{3\pi}{2}+k2\pi), k\in\mathbb{Z}$.

(h) $\log(3+\tan^2\frac{x}{2})-\log(1+\tan^2\frac{x}{2})+\frac{4}{\sqrt{3}}\arctan(\frac{1}{\sqrt{3}}\tan\frac{x}{2})$.

(i) $\cos x-\frac{5}{2}\arctan(\frac{1}{2}\cos x)$.

(j) $\frac{4}{\sqrt{3}}\arctan(\frac{1}{\sqrt{3}}\tan x)-x$ pro $x\in(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi), k\in\mathbb{Z}$.

Příklad 2.

(a) $\frac{1}{2}\log|\tan x-1|-\frac{1}{4}\log(\tan^2 x+1)+\frac{1}{2}x$ pro $\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{4}+k\pi\right),\left(\frac{\pi}{4}+k\pi,\frac{\pi}{2}+k\pi\right), k\in\mathbb{Z}$.

(b) $\frac{1}{6}\log|\sin^2-\sin x+1|-\frac{1}{3}\log(1+\sin x)+\frac{1}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}(2\sin x-1)\right)$.

(c) $x-\frac{1}{\sqrt{2}}\arctan(\sqrt{2}\tan x)$.

(d) $\frac{1}{\sqrt{5}}\arctan\left(\frac{1}{\sqrt{5}}(3\tan\frac{x}{2}+1)\right)$.

(e) $\log|2+\tan x|$.

Příklad 3.

(a) $-\log x-\log|\log x-1|$ pro $x\in(0,e)\cup(e,\infty)$.

(b) $-\frac{1}{2}x+\frac{1}{3}\log|e^x-1|+\frac{1}{6}\log(e^x+2)$ pro $x\in\mathbb{R}\setminus\{0\}$.

(c) $-2\sqrt{x}+4\sqrt[4]{x}-4\log(1+\sqrt[4]{x})$ pro $x>0$.

(d) $-x-4\sqrt{2x+3}-9\log|\sqrt{2x+3}-3|+\log(\sqrt{2x+3}+1)$ pro $x\in\left(-\frac{3}{2},-1\right),\cup(-1,3)\cup(3,\infty)$.

(e) $e^x+\frac{2}{3}\log|e^x-1|-\frac{1}{3}\log(e^{2x}+e^x+1)$ pro $x\in\mathbb{R}\setminus\{0\}$.

(f) $\log|\log(\log x)|$.

(g) $\log\left(1+\sqrt{\frac{x+2}{x-3}}\right)-\log\left|\sqrt{\frac{x+2}{x-3}}-1\right|-2\sqrt{\frac{2}{3}}\arctan\left(\sqrt{\frac{3}{2}}\sqrt{\frac{x+2}{x-3}}\right)$.

(h) $\frac{6}{7}\sqrt[6]{(x+1)^7}+\frac{6}{5}\sqrt[6]{(x+1)^5}-\frac{3}{2}\sqrt[6]{(x+1)^4}+2\sqrt[6]{(x+1)^3}-3\sqrt[6]{(x+1)^2}+6\sqrt[6]{x+1}-6\log(\sqrt[6]{x+1}+1)$.

(i) $\frac{1}{3}\log|\log x-1|-\frac{1}{6}\log|\log^2 x+\log x+1|-\frac{1}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}(\log x+\frac{1}{2})\right)$.

(j) $-\frac{3}{2}\log\left(1+\sqrt{\frac{x-1}{x+2}}\right)+\frac{3}{2}\log\left|1-\sqrt{\frac{x-1}{x+2}}\right|+\frac{3}{2}\cdot\frac{1}{1-\sqrt{\frac{x-1}{x+2}}}-\frac{3}{2}\cdot\frac{1}{1+\sqrt{\frac{x-1}{x+2}}}$.

(k) $\log\left|\sqrt{1-\frac{2}{x}}-1\right|-\log\left(\sqrt{1-\frac{2}{x}}+1\right)-\frac{1}{1+\sqrt{1-\frac{2}{x}}}+\frac{1}{1-\sqrt{1-\frac{2}{x}}}$.

(l) $6\log\sqrt[6]{x}-\frac{3}{2}\log(\sqrt[6]{x}+1)-\frac{9}{4}\log(2\sqrt[3]{x}-\sqrt[6]{x}+1)-\frac{3}{2\sqrt{7}}\arctan\left(\frac{4\sqrt[6]{x}-1}{\sqrt{7}}\right)$.

Příklad 4. Vzorové řešení: http://www.karlin.mff.cuni.cz/~kuncova/1920LS_MA2/16a_reseni.pdf

(a) $\frac{-2-4\sqrt[4]{x}}{(1+\sqrt[4]{x})^2}$ (př. 1(c))

(b) $2\arctan\sqrt{\frac{1-x}{1+x}}-\log\left|\frac{1+\sqrt{\frac{1-x}{1+x}}}{1-\sqrt{\frac{1-x}{1+x}}}\right|$ (pr. 2(c))

(c) $-\frac{2}{\sqrt{5}}\log\left|2\sqrt{\frac{2-x}{x+1}}+3+\sqrt{5}\right|+\frac{2}{\sqrt{5}}\log\left|-2\sqrt{\frac{2-x}{x+1}}-3+\sqrt{5}\right|-2\arctan\sqrt{\frac{2-x}{x+1}}$ (př. 3 (b))

(d) $\log\frac{\left(1+\sqrt{\frac{x+1}{x-1}}\right)^2}{1+\left(\sqrt{\frac{x+1}{x-1}}\right)^2}-\frac{2\sqrt{\frac{x+1}{x-1}}}{1+\left(\sqrt{\frac{x+1}{x-1}}\right)^2}$ (př. 2(a))

(e) $-\frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{x^2+x+1}-x+1+\sqrt{3}}{x-\sqrt{x^2+x+1}-1+\sqrt{3}}\right|$ (př. 3(c))

(f) $\frac{1}{2}\left(\sqrt{x^2+2x+4}-x+2\log\left|\sqrt{x^2+2x+4}-x-1\right|+\frac{3}{\sqrt{x^2+2x+4}-x-1}\right)$ (př. 3 (a))